ExB workshop
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Princeton NJ

Low-frequency plasma oscillations in Hall thrusters

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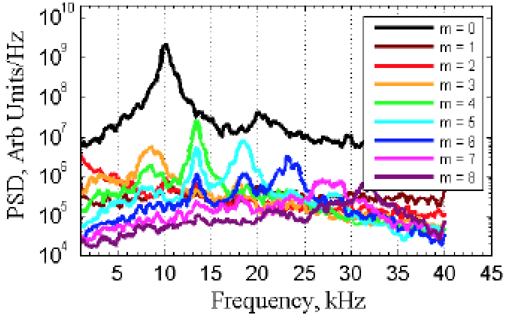
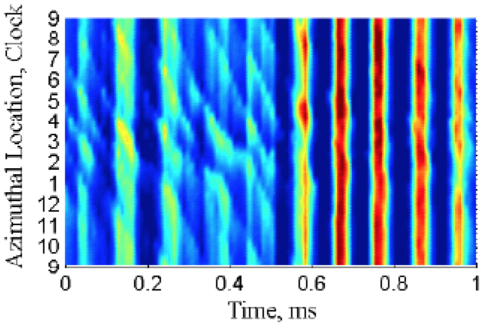
TWO TYPES OF BREATHING MODE? (STANDING VS. TRAVELING)

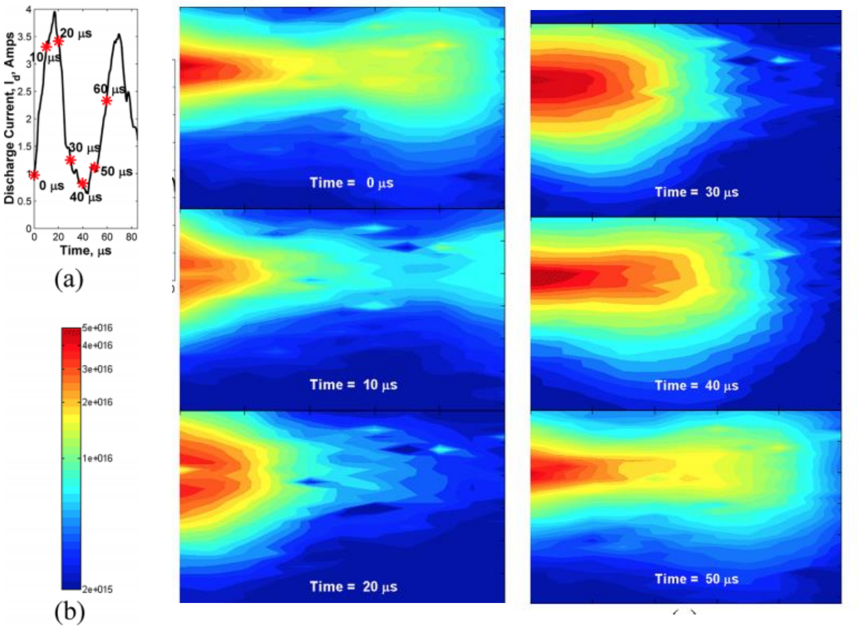
Sekerak breathing mode (300 V, 20 mg/s)





[Sekerak PhD 2014]

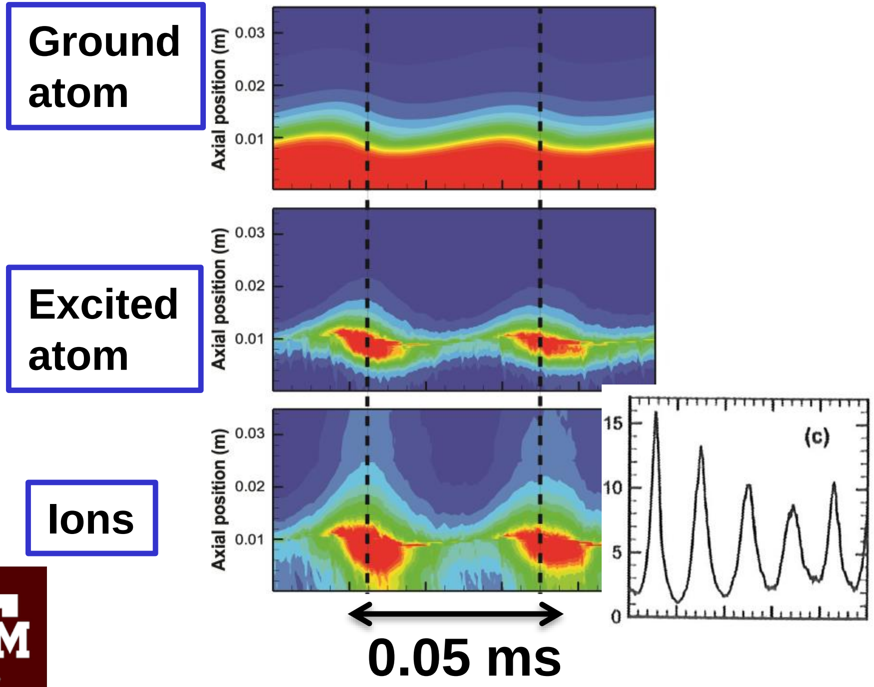
Lobbia Langmuir probe (200 V, 2A)



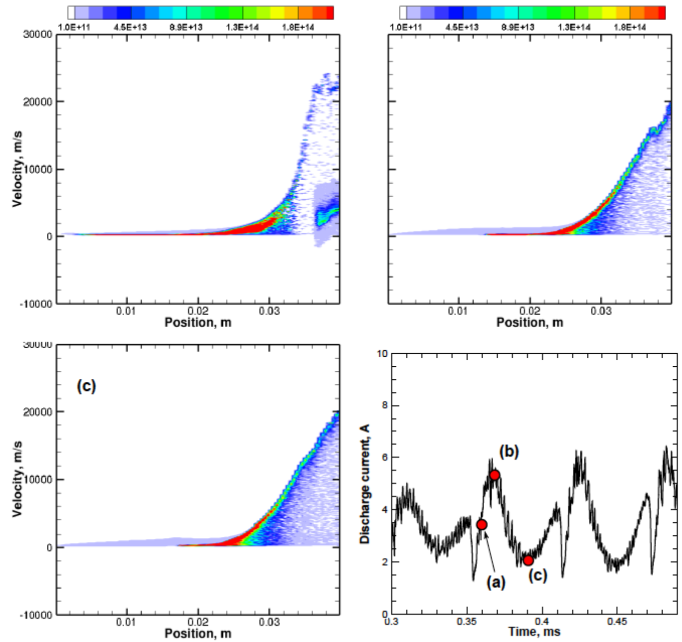
[Lobbia PhD 2010]

TWO TYPES OF BREATHING MODE? (STANDING VS. TRAVELING)

Standing wave?
(with high electron mobility)

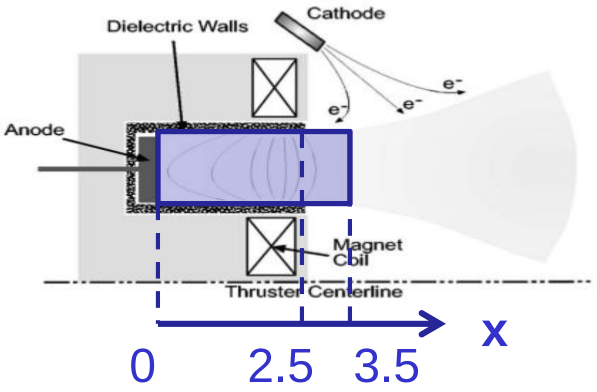


Traveling wave?
(with no anomalous mobility)



STANDING WAVE BREATHING MODE: 1D HYBRID DK-FLUID SIMULATION

- **Discharge channel of SPT-100 ML** [Gascon '03]
  - Peak magnetic field: 100 G – 200 G
  - Anode mass flow rate: 5.0 mg/s
  - Discharge voltage: 300V



- **Hybrid DK-Fluid model** [Hara, PoP 2012]
  - **Ion: direct kinetic (DK) model**
  - **Electron continuum model**
    - Electron: Drift-diffusion approximation (Ohm's law)
    - SEE depends on electron *thermal* energy (not *total* energy)
    - Anomalous mobility: two-region (inside channel: 1/160, outside: 1/16)
  - Electronically excited atoms + ground state atoms. Multispecies Hall thruster simulation with plasma reactions.

[Hara, Sekerak, Boyd, & Gallimore, JAP **115**, 203304 (2014)]

VALIDITY OF ELECTRON FLUID MODEL IN MAGNETIZED ELECTRONS

Mass $$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = \dot{n},$$

$S_{heat} = \boldsymbol{j} \cdot \boldsymbol{E}$, $S_{trans} = \boldsymbol{u} \cdot (\boldsymbol{R} - \nabla p)$,

Momentum $$\frac{\partial (mn\mathbf{u})}{\partial t} + \nabla \cdot \left(mn\mathbf{u}\mathbf{u} + \bar{\bar{P}} \right) = qn(\mathbf{E} + \mathbf{u} \times \mathbf{B}) + \mathbf{R},$$

Total energy $$\frac{\partial (n\epsilon)}{\partial t} + \nabla \cdot \left(n\epsilon\mathbf{u} + \mathbf{u} \cdot \bar{\bar{P}} + \mathbf{q} \right) = \mathbf{j} \cdot \mathbf{E} + S,$$

- $(mom.eq) \cdot \boldsymbol{u}$
- $K = \frac{1}{2} m|\boldsymbol{u}|^2 \approx \frac{1}{2} m u_\theta^2$

[Hara (unpublished)]

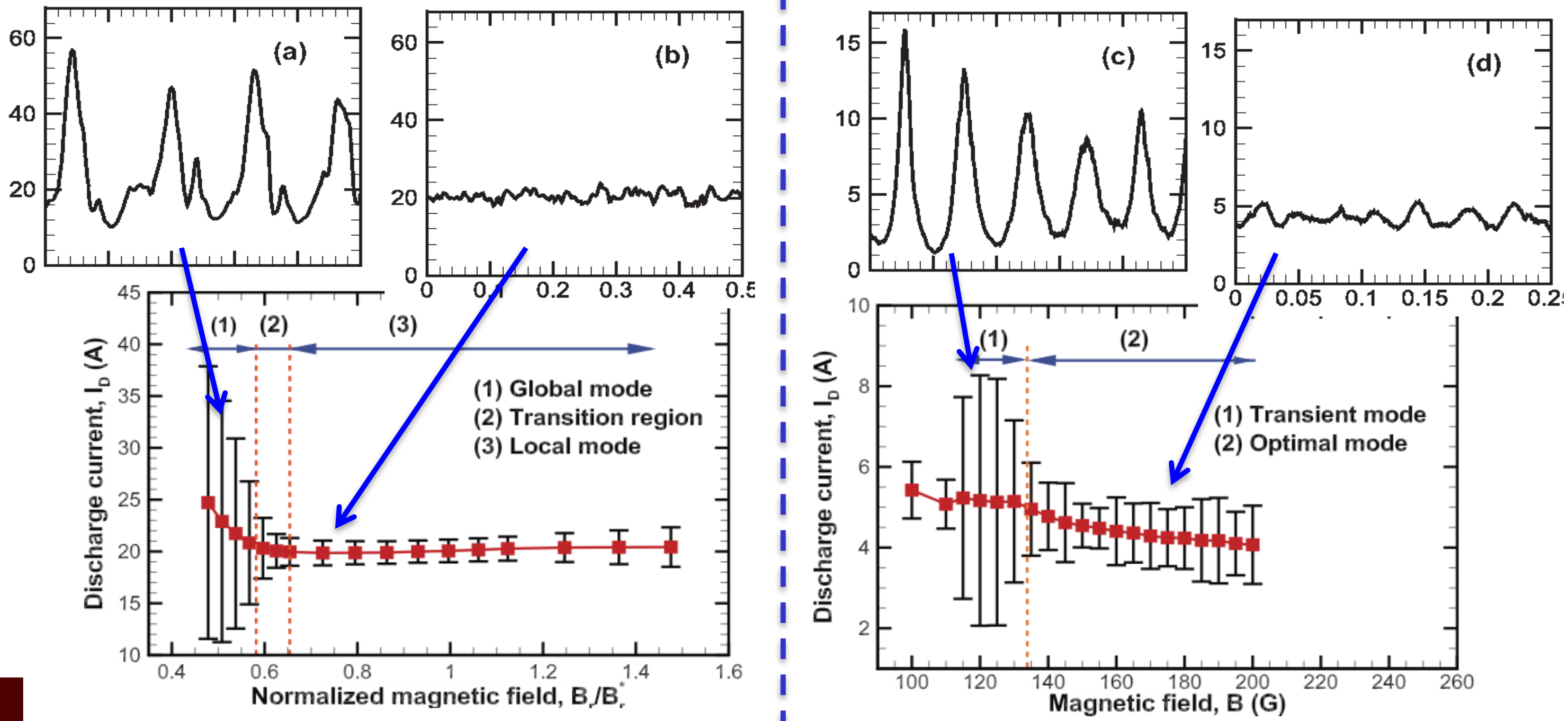
Kinetic energy $$\frac{\partial (nK)}{\partial t} + \nabla \cdot (nK\mathbf{u}) = \boxed{S_{\text{heat}}} - \boxed{S_{\text{trans}}} - nK\nu_{\text{ion}}$$

Internal energy $$\frac{\partial (ne)}{\partial t} + \nabla \cdot [(ne + p)\mathbf{u}] = \boxed{S_{\text{trans}}} + nK\nu_{\text{ion}} + S,$$

(i) If $K = 0$ (non-magnetized), $S_{heat} = S_{trans}$
Total energy input (j.E) goes to the internal energy (temperature).

(ii) If $K \neq 0$ (magnetized = drift!), $S_{heat} = \boldsymbol{j} \cdot \boldsymbol{E} \neq S_{trans}$
Recommended to solve the “total energy equation” and subtract the “kinetic energy” to get temperature.

PREDICTED SPT-100 SIMULATION RESULTS SHOW GOOD *QUALITATIVE* AGREEMENT WITH H6 EXPERIMENTS.

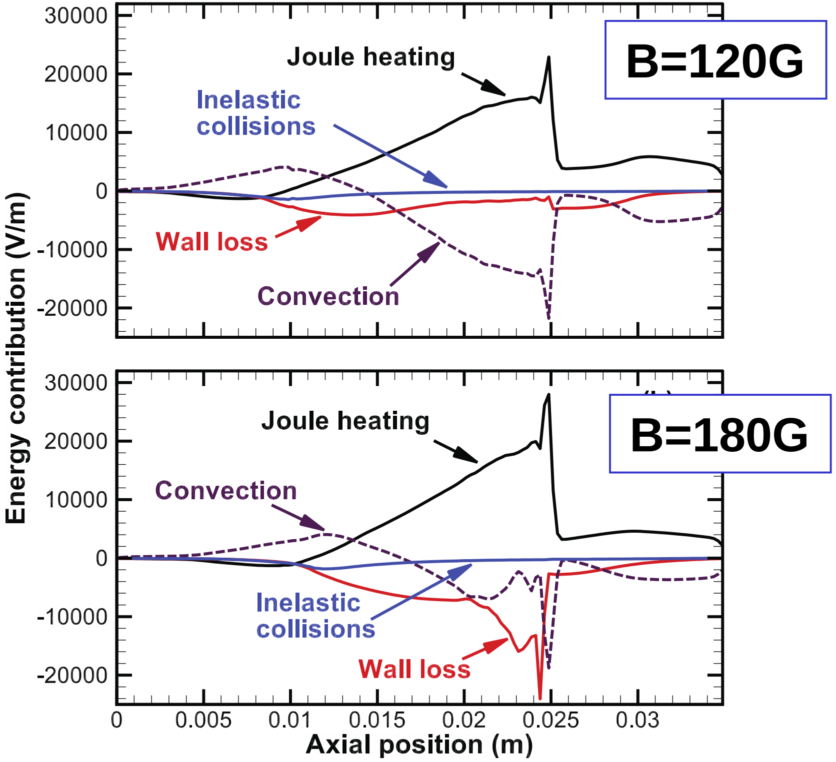


H6 experiment [Sekerak '14]

SPT-100 computation

STABILIZATION OF BREATHING MODE: ELECTRON HEAT FLUX

- Electron heating = **Joule heating**
- Electron cooling
  - **Oscillatory; B=120G**:
    - Small wall heat flux
    - Large convective heat flux
  - **Stable; B=180G**:
    - Large wall heat flux
    - Small convective heat flux
- **Electron heat transfer** plays an important role in “low frequency” ionization oscillations,



Relative energy transfer rate. Normalized by electron current density.

ELECTRON-PRESSURE COUPLING CAN INITIATE GRID-LEVEL OSCILLATIONS

- Nonmagnetized ion momentum equation (uncoupled)

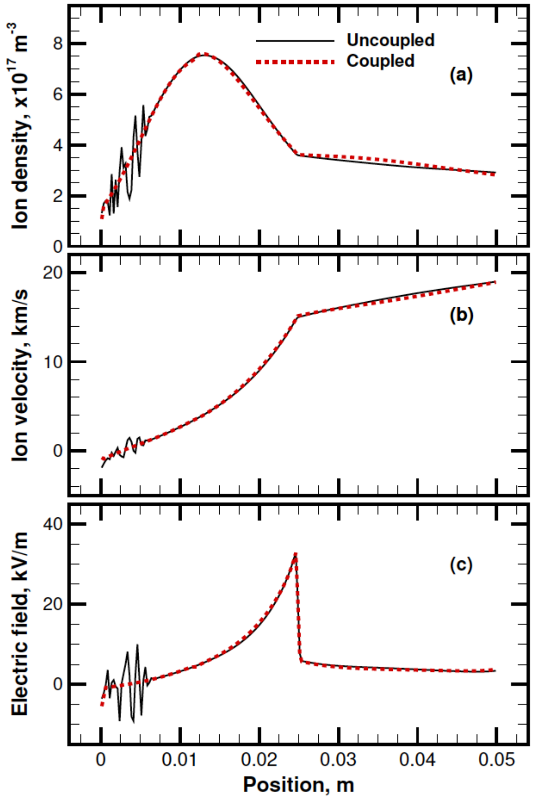
$$\frac{\partial(n_i \mathbf{u}_i)}{\partial t} + \nabla\left(n_i \mathbf{u}_i \mathbf{u}_i + \frac{p_i}{m_i}\right) = \frac{e}{m_i} n_i \mathbf{E},$$

- Using electron drift-diffusion model

$$\mathbf{E} = -\frac{\mathbf{u}_e}{\mu_\perp} - \frac{1}{e n_i}\nabla(n_i k_B T_e)$$

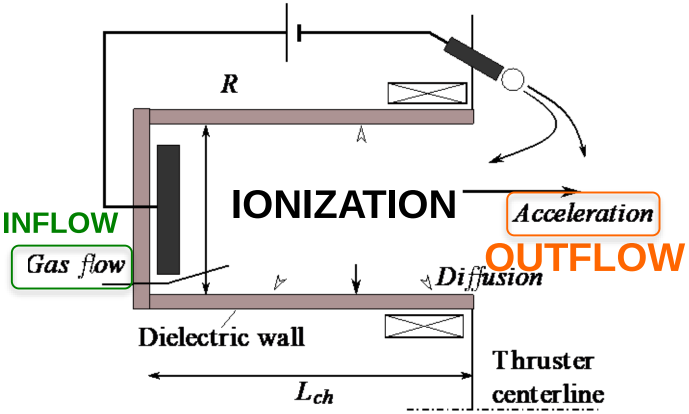
- Electron pressure coupled

$$\frac{\partial(n_i \mathbf{u}_i)}{\partial t} + \nabla\left(n_i \mathbf{u}_i \mathbf{u}_i + \frac{p_i + p_e}{m_i}\right) = -\frac{e n_i \mathbf{u}_e}{m_i \mu_\perp}.$$



[Hara (unpublished)]

PREDATOR-PREY TYPE MODEL
DAMPED OSCILLATION WITH NO TE FLUCTUATION



- **Neutral atoms:**

$$\frac{\partial [N_n]}{\partial t} = \boxed{-\frac{U_n}{L}[N_n]} \boxed{-[N_n][N_i]\xi_{ion}(T_e)} + \boxed{\frac{U_n}{L}[N_{\text{int}}]}$$

Inflow

- **Ions = electrons:**

$$\frac{\partial [N_i]}{\partial t} = \boxed{-\frac{U_i}{L}[N_i]} \boxed{+[N_n][N_i]\xi_{ion}(T_e)}$$

Outflow Ionization

Linear perturbation equation: $-\omega^2 + i\left(\frac{[N_{\text{int}}]}{[N_{n,0}]}\frac{U_n}{L}\right)\omega + [N_{n,0}][N_{i,0}]\xi_{ion}^2 = 0$

$$\omega_r = \pm\left(N_{i,0}N_{n,0}\xi_{ion}^2 - \gamma^2\right)^{\frac{1}{2}}$$

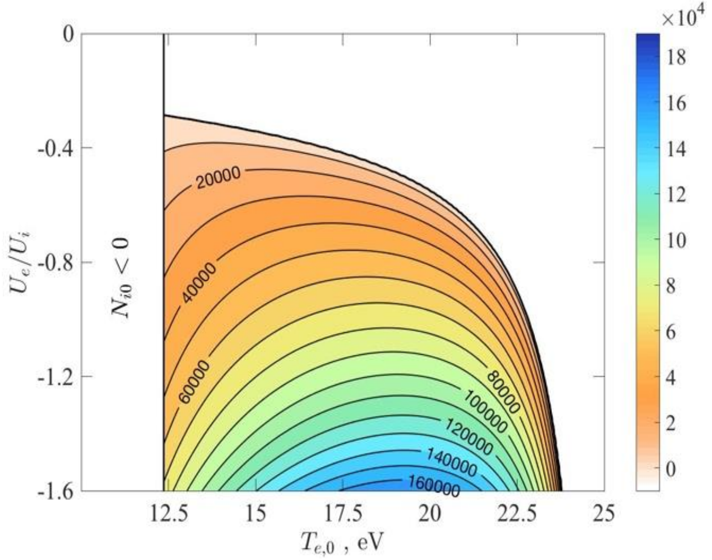
$$\gamma = -\frac{1}{2}\frac{N_{int}}{N_{int} - N_{n,0}}N_{i,0}\xi_{ion}.$$

Unconditionally damped with constant ξ_{ion} or T_e

STRATEGY FOR IONIZATION OSCILLATION THEORY

- **Minimum Te ($T_{e,min}$)** required for a steady-state discharge plasma

$$\underbrace{N_{int}\xi_{ion}(T_e)}_{\text{Maximum ion production}} > \underbrace{\frac{U_i}{L}}_{\text{Ion acceleration}},$$



Linear growth rate of ionization instability

$T_e < T_{e,min}$	**No steady-state ion density**	**Unstable mode**
$T_e >$ **$T_{e,min}$**	Steady-state ion density	Gamma<0 (stable)
		Gamma >0 **(ionization instability)**

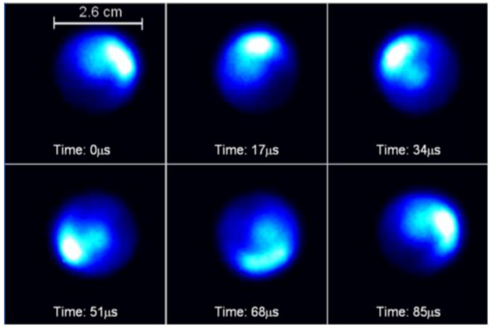
AZIMUTHALLY ROTATING SPOKES ON 2D Z-Θ SIMULATIONS

A quasi-neutral hybrid-PIC model

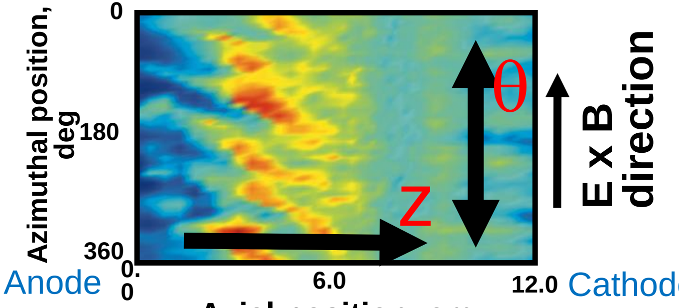
- Long-time calculation is required for full cylinder of channel (1-10 cm) to capture the "low-frequency" (~1 ms) phenomena
  - Full-PIC model is not feasible yet

$$\frac{\lambda_{\text{spoke}}}{\lambda_{\text{Debye}}} \sim 10^4 \quad \left(\frac{\omega_{\text{spoke}}}{\omega_{\text{plasma}}}\right)^{-1} \sim 10^6$$

- However, 2D electron fluid model for magnetized electrons is numerically challenging.[2]



Cylindrical Hall thruster (PPPL)



Simulation by Fernandez et al.[1]
Stable coherent structure was not captured

[1]Fernandez et al 2015; Lam 2015

[2]Hara and Boyd 2015.

PSEUDO-TIME STEPPING METHOD

- Finding “converged” solution to elliptic PDE is numerically challenging due to “ill-conditioned” matrix. Off-diagonal elements are dominant.

Elliptic PDE

$$\boxed{\nabla \cdot \left(n_\mathrm{e} \left[\mu\right] \nabla \phi - \left[\mu\right] \nabla \left(n_\mathrm{e} T_\mathrm{e}\right)\right) = n_\mathrm{e} \nu_\mathrm{ion}}$$

Discretized equation

$$\boxed{\begin{pmatrix} 1 & \cdots & \Omega_e \\ \vdots & \ddots & \vdots \\ -\Omega_e & \cdots & 1 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_N \end{pmatrix} = (RHS)}$$

- **Pseudo-time stepping method** of electron drift-diffusion model
  - Continuity equation

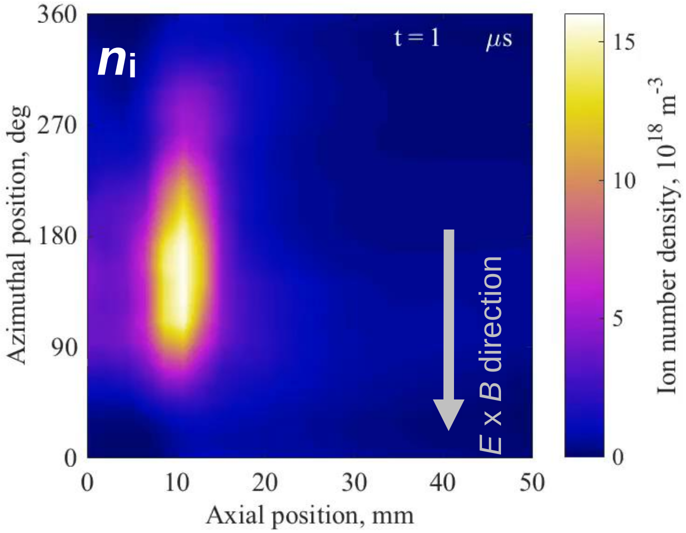
$$\cancel{\frac{n_\mathrm{e}}{T_\mathrm{e}} \frac{\partial \phi}{\partial t}} - \nabla \cdot \left(n_\mathrm{e} \vec{u}_\mathrm{e}\right) = -n_\mathrm{e} \nu_\mathrm{ion}$$

  - Momentum equation

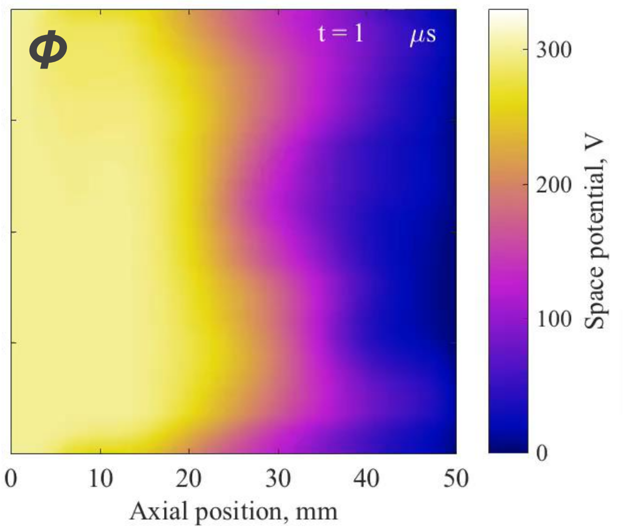
$$\cancel{\frac{1}{\nu_\mathrm{col}} \frac{\partial}{\partial t}} \left(n_\mathrm{e} \vec{u}_\mathrm{e}\right) - n_\mathrm{e} \left[\mu\right] \nabla \phi + \left[\mu\right] \nabla \left(n_\mathrm{e} T_\mathrm{e}\right) = -n_\mathrm{e} \vec{u}_\mathrm{e}$$

[Kawashima et al. J. Comp. Phys. (2015)] [Hara, Ph.D. UM (2015)]

“STABLE” LONG-TIME (>1 MS) SPOKE PROPAGATION



Ion number density distribution



Space potential distribution

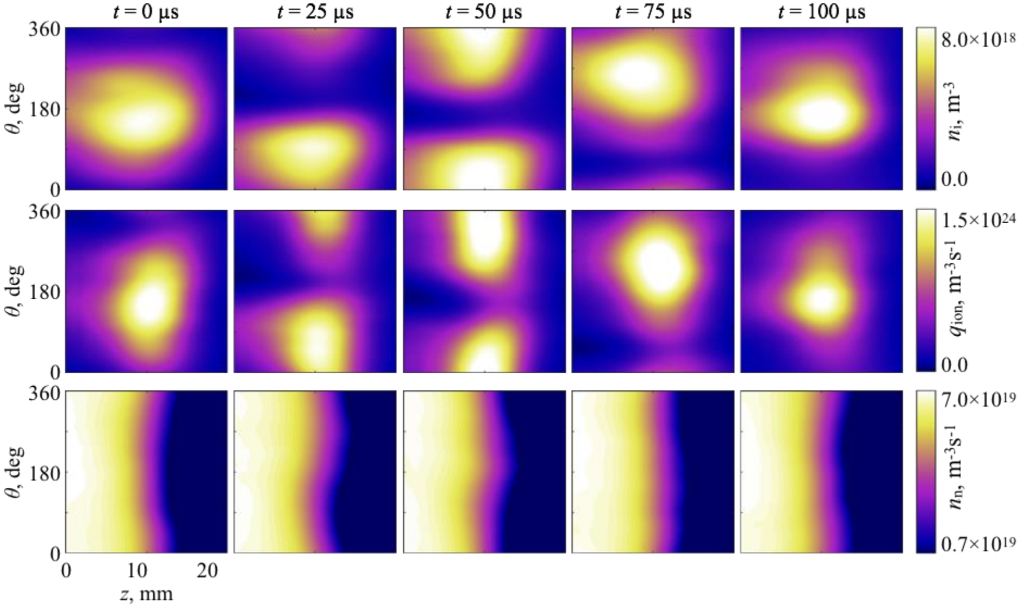
A stable simulation of azimuthally rotating spoke is achieved for over 1 ms

UNAMBIGUOUS SIGNAL OF ROTATING STRUCTURE

Ion density

Ionization rate

Neutral atom density



- A coherent structure (self-organization) rotating with m = 1 mode.
- Axial electron temperature profile is *quasi-static* in time. = No axial ionization oscillation (breathing mode; m = 0)

POSSIBLE MECHANISMS PROPOSED IN LITERATURE

1. **Esipchuk's Dispersion Relation[1] (1976)** $\nabla n_e,\ \nabla B$

$$\omega = k_z u_{i,z} - \frac{k_\perp^2 u_{i,z}^2}{2k_\theta (u_{e,\perp} - u_B)} \pm \frac{k_\perp u_{i,z}^2}{2(u_{e,\perp} - u_B)} \times \sqrt{1 + \left(\frac{k_z}{k_\theta}\right)^2 - 4\frac{k_z}{k_\theta}\frac{u_{e,\perp} - u_B}{u_{i,z}} + 4\frac{u_{e,\perp}(u_{e,\perp} - u_B)}{u_{i,z}^2}}$$

2. **Frias's Dispersion Relation[2] (2013)**

3. **Critical Ionization Velocity（CIV）**

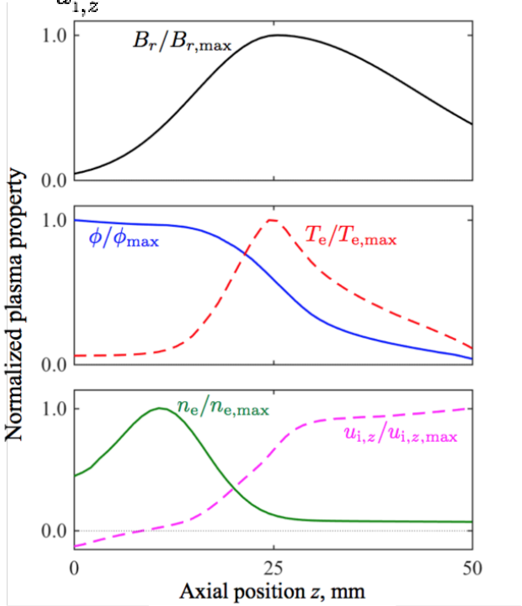
   - Propagation of ionization front (Kinetic energy of spoke = ionization energy), but no growth rate.[3]

$$v_{CIV} = \sqrt{\frac{2e\epsilon_{ion}}{m_i}}$$

4. **ElectroStatic Ion Cyclotron（ESIC）**

   - Phase velocity of RS was close to that of ion cyclotron oscillation in a 6 kW-level HET, but no growth rate,[4]

$$\omega^2 = k_\theta^2 u_s^2 + \omega_{c,i}^2$$



Azimuthally averaged axial results

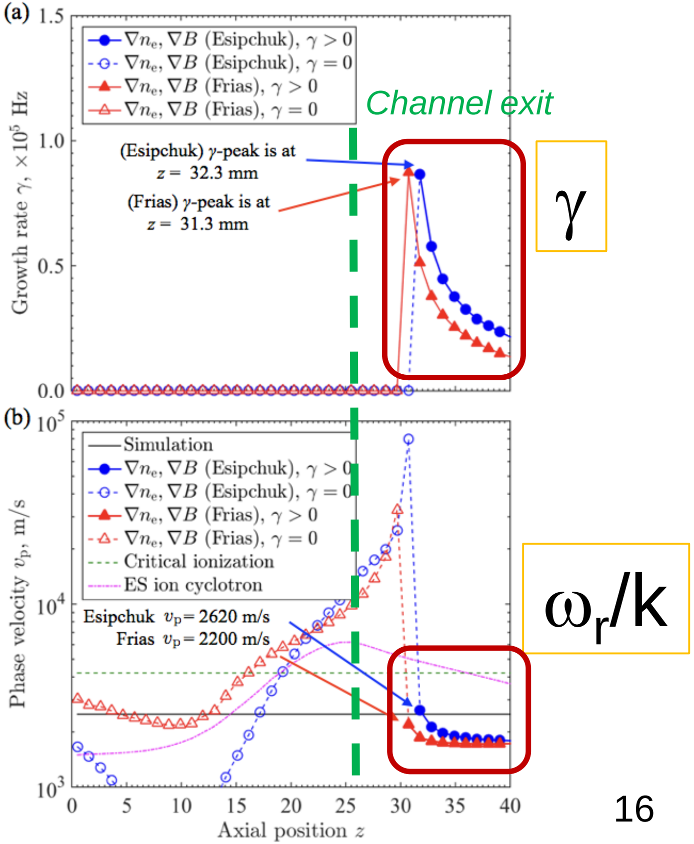
[1]Esipchuk and Tilinin, SPTP 21 (1976).
[2]Frias et al., PoP 19 (2013). [3]Boeuf, JAP 121 (2017).
[4]Sekerak, Univ. of Michigan Ph.D. Thesis (2014).

AGREEMENT WITH GRADIENT-DRIFT INSTABILITY THEORY

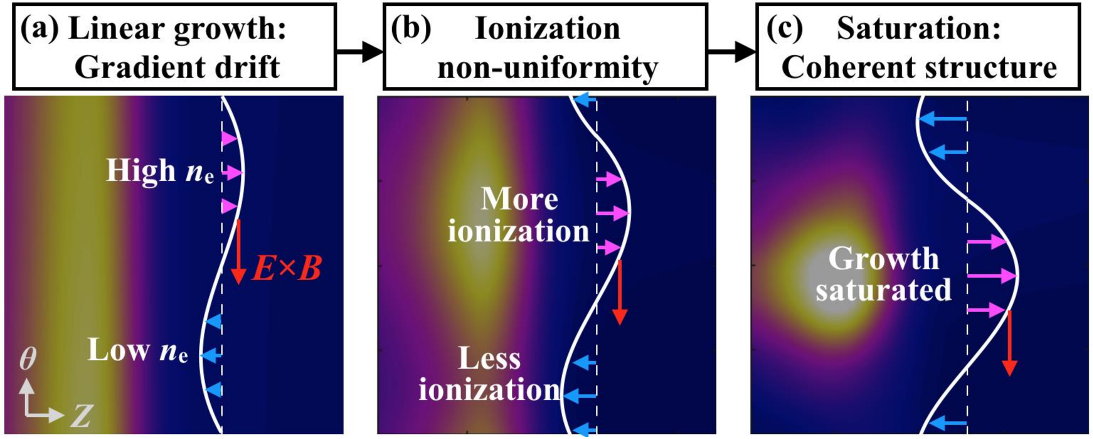
- 1D axial results (azimuthally averaged) are inserted into the dispersion relations.
- Large growth rate from **gradient drift instability** is observed in the downstream region.

Phase velocity comparison

	z, mm	v_p, m/s	Difference from Simulation
Simulation	–	2,500	–
Esipchuk	32.3	2,620	4.8%
Frias	31.3	2,200	12.0%
CIV	–	4,200	68.0%
ESIC	12.5	1,840	26.4%



HYPOTHESIS OF ROTATING SPOKE MECHANISM



1. Azimuthal perturbation (e.g., plasma density) grows based on linear instability of gradient drift wave depending on $\nabla n_e, \nabla B$ at downstream.
2. Ionization rate ($\dot{n_e} = n_e N_g k_{ion}$) becomes azimuthally nonuniform, then neutral atom (N_g) also becomes azimuthally nonuniform.
3. Saturated state (i.e., spoke) propagates with phase velocity of the largest growth rate of the instability.

CONCLUSION

- Breathing mode, axial ionization oscillation, is studied
- Azimuthally rotating spokes are observed using a quasineutral 2D hybrid fluid-PIC model
  - Numerical methods (e.g., fluid, kinetic, hybrid-kinetic) to be developed to capture these low-frequency oscillations
  - Various mechanisms to observe such low-frequency oscillation (e.g., cathode-coupling, wave growth model, convective instabilities)
  - Self-organization due to small scale phenomena plays an important role in plasma physics.